Fast Consensus and Metastability In a Highly Polarized Social Network

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Case study:
Rio de Janeiro's governor first-round election in 2018

Summary:

- The vote intention polls indicated a fierce dispute between Romário and Eduardo Paes.
- After counting the votes, an obscure candidate came before Romário in the first-round and eventually won the election.

Case study: 2018 Brazil Senate elections

- In the dispute for the Senate in Paraná, Minas Gerais and S. Paulo, Roberto Requião, Dilma Rousseff and Eduardo Suplicy, respectively, led comfortably in all vote intention polls.
- The three candidates were surprisingly defeated at the elections.

Case Study: 2018 Brazil Presidential Elections

- No vote intention poll predicted that Cabo Daciolo would be ahead of Henrique Meirelles and Marina Silva, nor that Alckmin would only have half of the votes estimated by previous polls.
- Finally and most importantly, polls did not predict candidate Bolsonaro's near-win in the first round.

What did we learn from exit polls?

- The exit poll carried out by IBOPE on the same day of the election predicted very precisely the officially reported results.
- The success of the exit poll suggests that the choice of cities and social groups in the sample design used by the polling organizations correctly reflected the diversity between regions and Brazilian social groups.
- It also indicates that the sample size was large enough to accurately predict the election result.

How to explain the error in the polls predictions?!

- On October 18, 2018, Folha de S. Paulo published an article by journalist Patrícia Campos Mello whose title was: **Entrepreneurs fund campaign against PT on WhatsApp**.
- News like this one suggests that social network campaigns were responsible for the discrepancies between intention polls and the officially reported results.
- The question is: may a WhatsApp campaign explain the errors in the intention-poll predictions?!

A wave a few days before the elections

- In October 2018, between the two rounds of the Brazilian elections, Mauro Paulino, director of DataFolha, wrote on his Twitter account:
- *"ELECTORAL SURVEYS showed the impulse of the wave in the final moments. RJ, MG and DF are clear examples. The discrepancies between the predictions suggested by Ibope and Datafolha polls a few days before elections and the officially reported results clearly show the wave."*

Number of social network users (2021)

Internet users who use a social network site at least once a month

1. China: 999.95 millions
2. Índia: 639.47 millions
3. USA: 295.48 millions
4. Indonesia: 193.43 millions
5. **Brazil**: 159.01 millions

www.statista.com

Number of social network users in Brazil (2021)

1. Facebook: 127 millions. 4th leading country.
2. Twitter: 19.05 millions. 4th leading country.
3. Whatsapp: 118.05 millions. 2nd leading country.

www.statista.com

A DataSenado opinion poll

- In 2019, the DataSenado Research Institute carried out an opinion poll.
- Among the survey participants, 45% said they had decided to vote during election period taking into account information seen in some social network.
- The most cited social networks were Facebook (31%) and Whatsapp (29%).

www.senado.leg.br

A wave days before the elections

- If Mauro Paulino is right, there was a sudden change in voting intentions in the last few days before the election.
- It was conjectured that this discrepancy was the result of social-media campaigning days before the elections.
- Verifying the plausibility of this conjecture is a scientific challenge.
- It is necessary to statistically model how a wave that changes the opinion of a large mass of voters in a very short time can occur in a social network.

A network with social pressure on actors

- It is natural to conjecture that the formation of a wave tending to unanimity is a consequence of the social pressure exerted on actors.
- To model this social pressure effect we will introduce a model in which each new opinion expressed by a social actor depends on the group's reactions to their last opinion issued.

How to model a social network?

- In this talk we will present a model that we have been developing in the last two years.
- It is a simple, flexible, easy to simulate mathematical model that has a realistic behavior.

The model has three main ingredients:

- A description of the set of interactions between social actors.
- A function describing how the opinions expressed by different members of the network affects the opinion and the activity rate of each specific member of the network.
- A parameter describing the level of polarization of the network.

A stochastic model for social networks

- Each social actor can express a “favorable” (+1) or “contrary” (-1) opinion on a certain subject.
- The social pressure on an actor determines the orientation and the rate at which he expresses opinions.
- When an actor expresses their opinion, social pressure on them is reset to 0,
- and simultaneously social pressure on the other actors is changed by one unit in the direction of the opinion that was just expressed.
- The network has a polarization coefficient that indicates the tendency of social actors to express an opinion in the same direction of the social pressure exerted on them.

A stochastic model for social networks

- $\mathcal{A} = \{1, 2, ..., N\}$: set of social actors.
- $\beta \in [0, +\infty)$: polarization coefficient of the network.
- $\mathcal{O} = \{+1, -1\}$: set of opinions.
- $O_n \in \mathcal{O}$: n-th opinion expressed in the network.
- $A_n \in \mathcal{A}$: actor that expressed the n-th opinion.
- $T_n \in (0, +\infty)$: time in which the n-th opinion was expressed.
- $T_0 = 0$.
- $U_t^{\beta}(a)$: social pressure on actor $a \in \mathcal{A}$ at time $t \in [0, +\infty)$.

A stochastic model for social networks

- The model describes the time evolution of the social pressure on the actors
- $$U_t^{\beta}(a) = \begin{cases} U_0^{\beta}(a), & \text{if } t < T_1,\ a \in \mathcal{A} \\ U_{T_{n-1}}^{\beta}(a) + O_n, & \text{if } t \in [T_n, T_{n+1}) \text{ and } a \neq A_n, \\ 0, & \text{if } t \in [T_n, T_{n+1}) \text{ and } a = A_n. \end{cases}$$
- In other terms, at time T_n, actor A_n expresses opinion O_n,
- and this changes the list of social pressures by
  - resetting to 0 the pressure on actor A_n
  - and by adding O_n to the pressure of all the other actors.

How T_n, A_n and O_n are chosen?

- The choices of A_n and O_n depends only on

$$U^{\beta}_{T_{n-1}} = (U^{\beta}_{T_{n-1}}(a) : a \in \mathcal{A}).$$

- $$\mathbb{P}(A_n = a, O_n = o \mid U^{\beta}_{T_{n-1}} = u) = \frac{e^{\beta o u(a)}}{\sum_{b \in \mathcal{A}} [e^{\beta u(b)} + e^{-\beta u(b)}]}.$$

- For any $s > 0$,

$$\mathbb{P}(T_n - T_{n-1} > s \mid U^{\beta}_{T_{n-1}} = u) = \exp\left(-s \sum_{b \in \mathcal{A}} [e^{\beta u(b)} + e^{-\beta u(b)}] \right).$$

A system of interacting point processes

- Call $n_1^a, n_2^a, \ldots$ the successive times in which actor a expresses an opinion and define
- $T_k^a = T_{n_k^a}, \quad O_k^a = O_{n_k^a}$.
- The social network can be described as a system of interacting marked point processes

$$\{((T_k^a, O_k^a) : k \geq 1) : a \in \mathcal{A}\}$$

  with memory of variable length!
- The time evolution of the system depends on U_0^β until the first time all the actors expressed an opinion at least once.
- This model belongs to the class of systems of interacting process with memory of variable length that was introduced by Galves and Löcherbach (2013).

A stochastic model for social networks

- $U_t^\beta = (U_t^\beta(a) : a \in \mathcal{A})$.
- $(U_t^\beta)_{t \geq 0}$ is a Markov jump process on

$$\mathcal{S} = \{u = (u(a) : a \in \mathcal{A}) \in \mathbb{Z}^N : \min\{|u(a)| : a \in \mathcal{A}\} = 0\},$$

- with infinitesimal generator

$$\mathcal{L}f(u) = \sum_{o \in \mathcal{O}} \sum_{b \in \mathcal{A}} \exp(\beta o u(b))[f(\pi^{b,o}(u) - f(u)],$$

  for any bounded function $f : \mathcal{S} \to \mathbb{R}$,
- where

$$\pi^{b,o}(u)(a) = \begin{cases} u(a) + o, \text{ if } a \neq b, \\ 0, \text{ if } a = b. \end{cases}$$

Heuristics: polarization coefficient and consensus

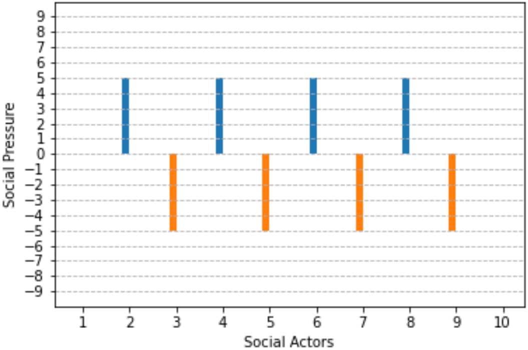
- Actors have the tendency to express opinions in the same direction of the social pressure the network exerts on them.
- The polarization coefficient tunes this effect.
- The greater the polarization coefficient, the faster the social network reaches **consensus**.
- By consensus we mean any list in which the social pressure of all actors have the same sign.
- Positive consensus:

$$\mathcal{C}^{+} = \{u \in \mathcal{S} : u \neq (0, ..., 0), u(a) \geq 0, \text{ for any } a \in \mathcal{A}\}.$$

- Negative consensus:

$$\mathcal{C}^{-} = \{u \in \mathcal{S} : u \neq (0, ..., 0), u(a) \leq 0, \text{ for any } a \in \mathcal{A}\}.$$

Simulating the time evolution of the social pressure



Highly polarized network

Highly polarized network (2)

Low polarized network

Proposition

$(U_t^\beta)_{t\geq 0}$ is an irreducible and positive-recurrent Markov process, converging exponentially fast to its invariant probability measure μ^β.

- **Sketch of the proof**: For any polarization coefficient $\beta \geq 0$ and for any $t > T_N$, we have the following bound:

$$\mathbb{P}(|U_t^\beta(a)| < N, \text{ for all } a \in \mathcal{A}) \geq \left(\frac{1}{N}\right)^N .$$

- We obtain this bound by considering the event in which for N consecutive instants, the actor that expressed an opinion is the actor with maximum social pressure at each instant.
- Since the social pressures are bounded, the transition rates are also bounded, and then, the expected first return time is bounded.

Concentration in the ladder lists when β increases

- $\mathcal{L}^+, \mathcal{L}^-$: set of positive and negative ladder lists, respectively.

$$\mathcal{L}^+ = \{u \in \mathcal{S} : \{u(1), ..., u(N)\} = \{0, 1, ..., N-1\}\},$$

$$\mathcal{L}^- = \{u \in \mathcal{S} : \{u(1), ..., u(N)\} = \{0, -1, ..., -(N-1)\}\}.$$

Theorem: There exists $C > 0$ such that for all $\beta \geq 1$,

$$\mu^\beta(\mathcal{L}^+ \cup \mathcal{L}^-) \geq 1 - Ce^{-\beta}.$$

Corollary: For any $u \in \mathcal{L}^+ \cup \mathcal{L}^-$,

$$\lim_{\beta \to +\infty} \mu^\beta(u) = \frac{1}{2N!}.$$

Fast consensus in a highly polarized network

- $(U_t^{\beta,u})_{t\geq 0}$: process with initial list $U_0^{\beta,u} = u$.
- For any $u \in \mathcal{S}$ and $B \subseteq \mathcal{S}$,

$$R^{\beta,u}(B) = \inf\{t \geq 0 : U_t^{\beta,u} \in B\}.$$

- $\vec{0} \in \mathcal{S}$: null list.
- **Theorem**: For any fixed $C' > 0$, fixed $\delta > 0$ and $u \in \mathcal{S} \setminus \{\vec{0}\}$,

$$\mathbb{P}\Big(\min\{R^{\beta,u}(\mathcal{L}^-), R^{\beta,u}(\mathcal{L}^+)\} > C' e^{-\beta(1-\delta)} \Big) \to 0 \text{ as } \beta \to \infty.$$

Metastability of the social network when $\beta \to +\infty$

Theorem: For any $u \in \mathcal{C}^+$ and $t \geq 0$,

$$\lim_{\beta \to +\infty} \mathbb{P}\left(\frac{R^{\beta,u}(\mathcal{L}^-)}{\mathbb{E}\big(R^{\beta,u}(\mathcal{L}^-)\big)} \geq t \right) = e^{-t}.$$

Fast consensus in a highly polarized network

- When β is large, with high probability
- the actor expressing a opinion is the one with highest social pressure in absolute value,
- and the expressed opinion on the network is on the same direction of the social pressure of the selected actor.
- In a highly polarized network the actor with higher in absolute value pressure speaks first.
- Starting with any list of pressures $u \in \mathcal{S}$, the system reaches a ladder list every time a sufficient long sequence of opinions expression is performed by actors with higher in absolute value social pressure.